Hadrons in Dense Resonance-Matter: A Chiral $SU(3)$ Approach

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Abstract

A nonlinear chiral SU(3) approach including the spin $\frac{3}{2}$ decuplet is developed to describe dense matter. The coupling constants of the baryon resonances to the scalar mesons are determined from the decuplet vacuum masses and SU(3) symmetry relations. Different methods of mass generation show significant differences in the properties of the spin-$\frac{3}{2}$ particles and in the nuclear equation of state.

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I. INTRODUCTION

The investigation of the equation of state of strongly interacting matter is one of the most challenging problems in nuclear and heavy ion physics. Dense nuclear matter exists in the interior of neutron stars, and its behaviour plays a crucial role for the structure and properties of these stellar objects. The behaviour of hadronic matter at high densities and temperatures strongly influences the observables in relativistic heavy ion collisions (e.g. flow, particle production,...). The latter depend on the bulk and nonequilibrium properties of the produced matter (e.g. pressure, density, temperature, viscosity,...) and the properties of the constituents (effective masses, decay widths, dispersion relations,...). So far it is not possible to determine the equation of state of hadronic matter at high densities (and temperatures) from first principles. QCD is not solvable in the regime of low momentum transfers and finite baryon densities. Therefore one has to pursue alternative ways to describe the hadrons in dense matter. Effective models, where only the relevant degrees of freedom for the problem are considered are solvable and can contain the essential characteristics of the full theory. For the case of strongly interacting matter this means that one considers hadrons rather than quarks and gluons as the relevant degrees of freedom. Several such models like the RMF model(QHD) and its extensions (QHD II, nonlinear Walecka model) successfully describe nuclear matter and finite nuclei [?,?,?,?,?]. Although these models are effective relativistic quantum field theories of baryons and mesons, they do not consider essential features of QCD, namely broken scale invariance and approximate chiral symmetry. Including SU(2) chiral symmetry in these models by adding repulsive vector mesons to the $SU(2)$-linear σ-model does neither lead to a reasonable description of nuclear matter ground state properties nor of finite nuclei [?]. Either one must use a nonlinear realization of chiral symmetry [?,?] or include a dilaton field and a logarithmic potential motivated by broken scale invariance [?,?] in order to obtain a satisfactory description of nuclear matter. Extending these approaches to the strangeness sector leads to a number of new, undetermined coupling constants due to the additional strange hadrons. Both to overcome this problem and to put restrictions on the coupling constants in the non-strange sector the inclusion of SU(3) [?] and chiral SU(3) [?,?] has been investigated in the last years. Recently [?] it was shown that an extended $SU(3) \times SU(3)$ chiral $\sigma - \omega$ model can describe nuclear matter ground state properties, vacuum properties and finite nuclei simultaneously. This model includes the lowest lying SU(3) multiplets of the baryons (octet), the spin-0 and the spin-1 mesons (nonets) as physical degrees of freedom. The present paper will discuss the predictions of this model for high density nuclear matter, including the spin $\frac{3}{2}$ baryon resonances (decuplet). This is necessary, because the increasing nucleonic fermi levels make the production of resonances energetically favorable at high densities. The paper is structured as follows: Section II summarizes the nonlinear chiral $SU(3) \times SU(3)$-model. Section III gives the baryon meson interaction, with main focus on the baryon meson-decuplet interaction and the constraints on the additional coupling constants. In section IV the resulting equations of motions and thermodynamic observables in the mean field approximation are discussed. Section V contains the results for dense hadronic matter, followed by the conclusions.

II. LAGRANGIAN OF THE NONLINEAR CHIRAL SU(3) MODEL

We use a relativistic field theoretical model of baryons and mesons based on chiral symmetry and scale invariance to describe strongly interacting nuclear matter. In earlier work the Lagrangian including the baryon octet, the spin-0 and spin-1 mesons has been developed [?]. Here the additional inclusion of the spin-$\frac{3}{2}$ baryon decuplet for infinite nuclear matter will be discussed. The general form of the Lagrangian then looks as follows:

$$\mathcal{L} = \mathcal{L}_{\mathrm{kin}} + \sum_{W=X,Y,V,\mathcal{A},u} \mathcal{L}_{\mathrm{BW}} + \mathcal{L}_{\mathrm{VP}} + \mathcal{L}_{\mathrm{vec}} + \mathcal{L}_0 + \mathcal{L}_{\mathrm{SB}}. \tag{1}$$

$\mathcal{L}_{\mathrm{kin}}$ is the kinetic energy term, $\mathcal{L}_{\mathrm{BW}}$ includes the interaction terms of the different baryons with the various spin-0 and spin-1 mesons. $\mathcal{L}_{\mathrm{VP}}$ contains the interaction terms of vector mesons with pseudoscalar mesons. $\mathcal{L}_{\mathrm{vec}}$ generates the masses of the spin-1 mesons through interactions with spin-0 mesons, and $\mathcal{L}_0$ gives the meson-meson interaction terms which induce the spontaneous breaking of chiral symmetry. It also includes the scale breaking logarithmic potential. Finally, $\mathcal{L}_{\rm SB}$ introduces an explicit symmetry breaking of the $\mathrm{U}(1)_A$ symmetry, the $\mathrm{SU}(3)_V$ symmetry, and the chiral symmetry. These terms have been discussed in detail in [?] and this shall not be repeated here. We will concentrate on the new terms in $\mathcal{L}_{\rm BW}$, which are due to adding the baryon resonances.

III. BARYON MESON INTERACTION

$\mathcal{L}_{BW}$ consists of the interaction terms of the included baryons (octet and decuplet) and the mesons (spin-0 and spin-1). For the spin-$\frac{1}{2}$ baryons the $SU(3)$ structure of the couplings to all mesons are the same, except for the difference in Lorentz space. For a general meson field W they read

$$\mathcal{L}_{\rm OW} = -\sqrt{2}g^W_{O8}\left(\alpha_{OW}[\overline{B}\mathcal{O}BW]_F + (1-\alpha_{OW})[\overline{B}\mathcal{O}BW]_D\right) - g^W_{O1}\frac{1}{\sqrt{3}}\mathrm{Tr}(\overline{B}\mathcal{O}B)\mathrm{Tr}W\,, \quad (2)$$

with $[\overline{B}\mathcal{O}BW]_F := \mathrm{Tr}(\overline{B}\mathcal{O}WB - \overline{B}\mathcal{O}BW)$ and $[\overline{B}\mathcal{O}BW]_D := \mathrm{Tr}(\overline{B}\mathcal{O}WB + \overline{B}\mathcal{O}BW) - \frac{2}{3}\mathrm{Tr}(\overline{B}\mathcal{O}B)\mathrm{Tr}W$. The different terms to be considered are those for the interaction of spin-$\frac{1}{2}$ baryons (B), with scalar mesons ($W = X, \mathcal{O} = 1$), with vector mesons ($W = V_\mu, \mathcal{O} = \gamma_\mu$), with axial vector mesons ($W = \mathcal{A}_\mu, \mathcal{O} = \gamma_\mu\gamma_5$) and with pseudoscalar mesons ($W = u_\mu, \mathcal{O} = \gamma_\mu\gamma_5$), respectively. For the spin-$\frac{3}{2}$ baryons (D^μ) one can construct a coupling term similar to (2)

$$\mathcal{L}_{\rm DW} = -\sqrt{2}g^W_{D8}[\overline{D^\mu}\mathcal{O}D_\mu W] - g^W_{D1}[\overline{D^\mu}\mathcal{O}D_\mu]TrW\,, \quad (3)$$

where $[\overline{D^\mu}\mathcal{O}D_\mu W]$ and $[\overline{D^\mu}\mathcal{O}D_\mu]$ are obtained from coupling $[\bar{10}]\times[10]\times[8] = [1]+[8]+[27]+[64]$ and $[\bar{10}]\times[10]\times[1]$ to an SU(3) singlet, respectively. In the following we focus on the couplings of the baryons to the scalar mesons which dynamically generate the hadron masses and vector mesons which effectively describe the short-range repulsion. For the pseudoscalar mesons only a pseudovector coupling is possible, since in the nonlinear realization of chiral symmetry [?] they only appear in derivative terms. Pseudoscalar and axial mesons have a vanishing expectation value at the mean field level, so that their coupling terms will not be discussed in detail here.

Scalar Mesons

The baryons and the scalar mesons transform equally in the left and right subspace. Therefore, in contrast to the linear realization of chiral symmetry, an f-type coupling is allowed for the baryon-octet-meson interaction. In addition, it is possible to construct mass terms for baryons and to couple them to chiral singlets. Since the current quark masses in QCD are small compared to the hadron masses, we will use baryonic mass terms only as small corrections to the dynamically generated masses. Furthermore a coupling of the baryons to the dilaton field χ is also possible, but this will be discussed in a later publication. After insertion of the vacuum matrix $\langle X \rangle$, (Eq.A4), one obtains the baryon masses as generated by the vacuum expectation value (VEV) of the two meson fields:

$$\begin{aligned} m_N &= m_0 - \frac{1}{3} g^S_{O8}(4\alpha_{OS} - 1)(\sqrt{2}\zeta - \sigma) \\ m_\Lambda &= m_0 - \frac{2}{3} g^S_{O8}(\alpha_{OS} - 1)(\sqrt{2}\zeta - \sigma) \\ m_\Sigma &= m_0 + \frac{2}{3} g^S_{O8}(\alpha_{OS} - 1)(\sqrt{2}\zeta - \sigma) \\ m_\Xi &= m_0 + \frac{1}{3} g^S_{O8}(2\alpha_{OS} + 1)(\sqrt{2}\zeta - \sigma) \end{aligned} \tag{4}$$

with $m_0 = g^S_{O1}(\sqrt{2}\sigma + \zeta)/\sqrt{3}$. The parameters g^S_{O1}, g^S_{O8} and α_{OS} can be used to fit the baryon-octet masses to their experimental values. Besides the current quark mass terms discussed in [**?**], no additional explicit symmetry breaking term is needed. Note that the nucleon mass depends on the *strange condensate* ζ*!* For $\zeta = \sigma/\sqrt{2}$ (i.e. $f_\pi = f_K$), the masses are degenerate, and the vacuum is SU(3)$_V$-invariant. For the spin-$\frac{3}{2}$ baryons the procedure is similar. If the vacuum matrix for the scalar condensates is inserted one obtains the dynamically generated vacuum masses of the baryon decuplet

$$m_\Delta = g^S_D \left[(3 - \alpha_{DS})\sigma + \alpha_{DS}\sqrt{2}\zeta\right] \tag{5}$$

$$
\begin{aligned}
m_{\Sigma^*} &= g_D^S\left[2\sigma+\sqrt{2}\zeta\right]\\
m_{\Xi^*} &= g_D^S\left[(1+\alpha_{DS})\sigma+(2-\alpha_{DS})\sqrt{2}\zeta\right]\\
m_{\Omega} &= g_D^S\left[2\alpha_{DS}\sigma+(3-\alpha_{DS})\sqrt{2}\zeta\right]
\end{aligned}
$$

The new parameters are connected to the parameters in (3) by $g_{D8}^W=-\sqrt{120}(1-\alpha_{DS})g_D^S$ and $g_{D1}^W=\sqrt{90}g_D^S$. g_D^S and α_{DS} can now be fixed to reproduce the masses of the baryon decuplet. As in the case of the nucleon, the coupling of the Δ to the strange condensate is nonzero.

It is desirable to have an alternative way of baryon mass generation, where the nucleon and the Δ mass depend only on σ. For the nucleon this can be accomplished for example by taking the limit $\alpha_{OS}=1$ and $g_{O1}^S=\sqrt{6}g_{O8}^S$. Then, the coupling constants between the baryon octet and the two scalar condensates are related to the additive quark model. This leaves only one coupling constant to adjust for the correct nucleon mass. For a fine-tuning of the remaining masses, it is necessary to introduce an explicit symmetry breaking term, that breaks the SU(3)-symmetry along the hypercharge direction. A possible term already discussed in [**?**,**?**], which respects the Gell-Mann-Okubo mass relation, is

$$
\mathcal{L}_{\Delta m}=-m_1\mathrm{Tr}(\overline{B}B-\overline{B}BS)-m_2\mathrm{Tr}(\overline{B}SB), \tag{6}
$$

where $S_b^a=-\frac{1}{3}[\sqrt{3}(\lambda_8)_b^a-\delta_b^a]$. As in the first case, only three coupling constants, $g_{N\sigma}\equiv 3g_{O8}^S$, m_1 and m_2, are sufficient to reproduce the experimentally known baryon masses. Explicitly, the baryon masses have the values

$$
\begin{aligned}
m_N &= -g_{N\sigma}\sigma \qquad\qquad (7)\\
m_\Xi &= -\frac{1}{3}g_{N\sigma}\sigma-\frac{2}{3}g_{N\sigma}\sqrt{2}\zeta+m_1+m_2\\
m_\Lambda &= -\frac{2}{3}g_{N\sigma}\sigma-\frac{1}{3}g_{N\sigma}\sqrt{2}\zeta+\frac{m_1+2m_2}{3}\\
m_\Sigma &= -\frac{2}{3}g_{N\sigma}\sigma-\frac{1}{3}g_{N\sigma}\sqrt{2}\zeta+m_1,
\end{aligned}
$$

For the baryon decuplet the choice $\alpha_{DS}=0$ yields coupling constants related to the additive quark model. We introduce an explicit symmetry breaking proportional to the number of strange quarks for a given baryon species. Here we need only one additional parameter m_{Ds} to obtain the masses of the baryon decuplet:

$$\begin{aligned}
m_{\Delta} &= g_{\Delta\sigma}\,[3\sigma] \\
m_{\Sigma^*} &= g_{\Delta\sigma}\left[2\sigma+\sqrt{2}\zeta\right]+m_{Ds} \\
m_{\Xi^*} &= g_{\Delta\sigma}\left[1\sigma+2\sqrt{2}\zeta\right]+2m_{Ds} \\
m_{\Omega} &= g_{\Delta\sigma}\left[0\sigma+3\sqrt{2}\zeta\right]+3m_{Ds}
\end{aligned} \tag{8}$$

For both versions of the baryon-meson interaction the parameters are fixed to yield the baryon masses of the octet and the decuplet. The corresponding parameter set C_2, has been discussed in detail in [**?**].

Vector mesons

For the spin-$\frac{1}{2}$ baryons two independent interaction terms with spin-1 mesons can be constructed, in analogy to the interaction of the baryon octet with the scalar mesons. They correspond to the antisymmetric (f-type) and symmetric (d-type) couplings, respectively. From the universality principle [**?**] and the vector meson dominance model one may conclude that the d-type coupling should be small. Here $\alpha_V = 1$, i.e. pure f-type coupling, is used. It was shown in [**?**], that a small admixture of d-type coupling allows for some fine-tuning of the single-particle energy levels of nucleons in nuclei. As in the case of scalar mesons, for $g^V_{O1} = \sqrt{6}g^V_{O8}$, the strange vector field $\phi_\mu \sim \bar{s}\gamma_\mu s$ does not couple to the nucleon. The remaining couplings to the strange baryons are then determined by symmetry relations:

$$\begin{aligned}
g_{N\omega} &= (4\alpha_V-1)g^V_{O8} & & \\
g_{\Lambda\omega} &= \frac{2}{3}(5\alpha_V-2)g^V_{O8} & g_{\Lambda\phi} &= -\frac{\sqrt{2}}{3}(2\alpha_V+1)g^V_{O8} \\
g_{\Sigma\omega} &= 2\alpha_V g^V_{O8} & g_{\Sigma\phi} &= -\sqrt{2}(2\alpha_V-1)g^V_{O8} \\
g_{\Xi\omega} &= (2\alpha_V-1)g^V_{O8} & g_{\Xi\phi} &= -2\sqrt{2}\alpha_V g^V_{O8} \quad .
\end{aligned} \tag{9}$$

In the limit $\alpha_V = 1$, the relative values of the coupling constants are related to the additive quark model via:

$$g_{\Lambda\omega} = g_{\Sigma\omega} = 2g_{\Xi\omega} = \frac{2}{3}g_{N\omega} = 2g_{O8}^{V} \qquad g_{\Lambda\phi} = g_{\Sigma\phi} = \frac{g_{\Xi\phi}}{2} = \frac{\sqrt{2}}{3}g_{N\omega}. \tag{10}$$

Note that all coupling constants are fixed once e.g. $g_{N\omega}$ is specified. For the coupling of the baryon resonances to the vector mesons we obtain the same Clebsch-Gordan coefficients as for the coupling to the scalar mesons. This leads to the following relations between the coupling constants:

$$\begin{aligned}
g_{\Delta\omega} &= (3-\alpha_{DV})g_{DV} & g_{\Delta\phi} &= \sqrt{2}\alpha_{DV}g_{DV} \\
g_{\Sigma^*\omega} &= 2g_{DV} & g_{\Sigma^*\phi} &= \sqrt{2}g_{DV} \\
g_{\Xi^*\omega} &= (1+\alpha_{DV})g_{DV} & g_{\Xi^*\phi} &= \sqrt{2}(2-\alpha_{DV})g_{DV} \\
g_{\Omega\omega} &= \alpha_{DV}g_{DV} & g_{\Omega\phi} &= \sqrt{2}(3-\alpha_{DV})g_{DV} \quad .
\end{aligned} \tag{11}$$

In analogy to the octet case we set $\alpha_{DV} = 0$, so that the strange vector meson ϕ does not couple to the Δ-baryon. The resulting coupling constants again obey the additive quark model constraints:

$$\begin{aligned}
g_{\Delta\omega} &= \frac{3}{2}g_{\Sigma^*\omega} = 3g_{\Xi^*\omega} = 3g_{DV} \qquad g_{\Omega\omega} = 0 \\
g_{\Omega\phi} &= \frac{3}{2}g_{\Xi^*\phi} = 3g_{\Sigma^*\phi} = \sqrt{2}g_{\Delta\omega} \qquad g_{\Delta\phi} = 0
\end{aligned} \tag{12}$$

Hence all coupling constants of the baryon decuplet are again fixed if one overall coupling g_{DV} is specified. Since there is no vacuum restriction on the Δ-ω coupling, like in the case of the scalar mesons, we have to consider different constraints. This will be discussed in section V.

IV. MEAN-FIELD APPROXIMATION

The terms discussed so far involve the full quantum field operators. They cannot be treated exactly. Hence, to investigate hadronic matter properties at finite baryon density we adopt the mean-field approximation. This nonperturbative relativistic method is applied to solve approximately the nuclear many body problem by replacing the quantum field operators by their classical expectation values (for a recent review see [?]), i.e. the fluctuations around the vacuum expectation values of the field operators are neglected:

$$\sigma(x) = \langle\sigma\rangle + \delta\sigma \to \langle\sigma\rangle \equiv \sigma\,; \quad \zeta(x) = \langle\zeta\rangle + \delta\zeta \to \langle\zeta\rangle \equiv \zeta \tag{13}$$
$$\omega_\mu(x) = \langle\omega\rangle\delta_{0\mu} + \delta\omega_\mu \to \langle\omega_0\rangle \equiv \omega\,; \quad \phi_\mu(x) = \langle\phi\rangle\delta_{0\mu} + \delta\phi_\mu \to \langle\phi_0\rangle \equiv \phi.$$

The fermions are treated as quantum mechanical single-particle operators. The derivative terms can be neglected and only the time-like component of the vector mesons $\omega \equiv \langle\omega_0\rangle$ and $\phi \equiv \langle\phi_0\rangle$ survive if we assume homogeneous and isotropic infinite baryonic matter. Additionally, due to parity conservation we have $\langle\pi_i\rangle = 0$. The baryon resonances are treated as spin-$\frac{1}{2}$ particles with spin-$\frac{3}{2}$ degeneracy. After these approximations the Lagrangian (1) reads

$$\begin{aligned}
\mathcal{L}_{BM} + \mathcal{L}_{BV} &= -\sum_i \overline{\psi_i}[g_{i\omega}\gamma_0\omega^0 + g_{i\phi}\gamma_0\phi^0 + m_i^*]\psi_i \\
\mathcal{L}_{vec} &= \frac{1}{2}m_\omega^2\frac{\chi^2}{\chi_0^2}\omega^2 + \frac{1}{2}m_\phi^2\frac{\chi^2}{\chi_0^2}\phi^2 + g_4^4(\omega^4 + 2\phi^4) \\
\mathcal{V}_0 &= \frac{1}{2}k_0\chi^2(\sigma^2+\zeta^2) - k_1(\sigma^2+\zeta^2)^2 - k_2(\frac{\sigma^4}{2} + \zeta^4) - k_3\chi\sigma^2\zeta \\
&\quad + k_4\chi^4 + \frac{1}{4}\chi^4\ln\frac{\chi^4}{\chi_0^4} - \frac{\delta}{3}\ln\frac{\sigma^2\zeta}{\sigma_0^2\zeta_0} \\
\mathcal{V}_{SB} &= \left(\frac{\chi}{\chi_0}\right)^2\left[m_\pi^2 f_\pi\sigma + (\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi)\zeta\right],
\end{aligned}$$

with the effective mass m_i^* of the baryon i, which is defined according to section III for $i = N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega$.

Now it is straightforward to write down the expression for the thermodynamical potential of the grand canonical ensemble, Ω, per volume V at a given chemical potential μ and at zero temperature:

$$\frac{\Omega}{V} = -\mathcal{L}_{vec} - \mathcal{L}_0 - \mathcal{L}_{SB} - \mathcal{V}_{vac} - \sum_i \frac{\gamma_i}{(2\pi)^3}\int d^3k[E_i^*(k) - \mu_i^*] \tag{14}$$

The vacuum energy $\mathcal{V}_{vac}$ (the potential at $\rho = 0$) has been subtracted in order to get a vanishing vacuum energy. The γ_i denote the fermionic spin-isospin degeneracy factors. The single particle energies are $E_i^*(k) = \sqrt{k_i^2 + m_i^{*2}}$ and the effective chemical potentials read $\mu_i^* = \mu_i - g_{\omega i}\omega - g_{\phi i}\phi$.

The mesonic fields are determined by extremizing $\frac{\Omega}{V}(\mu, T=0)$:

$$\frac{\partial(\Omega/V)}{\partial\chi} = -\omega^2 m_\omega^2 \frac{\chi}{\chi_0^2} + k_0\chi(\sigma^2+\zeta^2) - k_3\sigma^2\zeta + \left(4k_4 + 1 + 4\ln\frac{\chi}{\chi_0} - 4\frac{\delta}{3}\ln\frac{\sigma^2\zeta}{\sigma_0^2\zeta_0}\right)\chi^3 + \tag{15}$$
$$+ 2\frac{\chi}{\chi_0^2}\left[m_\pi^2 f_\pi \sigma + (\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi)\zeta\right] = 0$$

$$\frac{\partial(\Omega/V)}{\partial\sigma} = k_0\chi^2\sigma - 4k_1(\sigma^2+\zeta^2)\sigma - 2k_2\sigma^3 - 2k_3\chi\sigma\zeta - 2\frac{\delta\chi^4}{3\sigma} + \tag{16}$$
$$+ \left(\frac{\chi}{\chi_0}\right)^2 m_\pi^2 f_\pi + \sum_i \frac{\partial m_i^*}{\partial\sigma}\rho_i^s = 0$$

$$\frac{\partial(\Omega/V)}{\partial\zeta} = k_0\chi^2\zeta - 4k_1(\sigma^2+\zeta^2)\zeta - 4k_2\zeta^3 - k_3\chi\sigma^2 - \frac{\delta\chi^4}{3\zeta} + \tag{17}$$
$$+ \left(\frac{\chi}{\chi_0}\right)^2\left[\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right] + \sum_i \frac{\partial m_i^*}{\partial\zeta}\rho_i^s = 0$$

$$\frac{\partial(\Omega/V)}{\partial\omega} = -\left(\frac{\chi}{\chi_0}\right) m_\omega^2\omega - 4g_4^4\omega^3 + \sum_i \frac{g_{i\omega}}{\rho_i} = 0 \tag{18}$$

$$\frac{\partial(\Omega/V)}{\partial\phi} = -\left(\frac{\chi}{\chi_0}\right) m_\phi^2\phi - 8g_4^4\phi^3 + \sum_i \frac{g_{i\phi}}{\rho_i} = 0 \tag{19}$$

The scalar densities ρ_i^s and the vector densities ρ_i can be calculated analytically for the case $T=0$, yielding

$$\rho_i^s = \gamma_i \int \frac{d^3k}{(2\pi)^3}\frac{m_i^*}{E_i^*} = \frac{\gamma_i m_i^*}{4\pi^2}\left[k_{Fi}E_{Fi}^* - m_i^{*2}\ln\left(\frac{k_{Fi}+E_{Fi}^*}{m_i^*}\right)\right] \tag{20}$$
$$\rho_i = \gamma_i \int_0^{k_{Fi}} \frac{d^3k}{(2\pi)^3} = \frac{\gamma_i k_{Fi}^3}{6\pi^2}\,. \tag{21}$$

The energy density and the pressure follow from the Gibbs–Duhem relation, $\epsilon = \Omega/V + \sum_i \mu_i\rho^i$ and $p = -\Omega/V$. The Hugenholtz–van Hove theorem [?] yields the Fermi surfaces as $E^*(k_{Fi}) = \sqrt{k_{Fi}^2 + m_i^{*2}} = \mu_i^*$.

V. RESULTS FOR DENSE NUCLEAR MATTER

A. Parameters

Fixing of the parameters to vacuum and nuclear matter ground state properties was discussed in detail in [?]. It has been shown that the obtained parameter sets describe the nuclear matter saturation point, hadronic vacuum masses and properties of finite nuclei reasonably well. The additional parameters here are the couplings of the baryon resonances to the scalar and vector mesons. For the scalar mesons this is done by a fit to the vacuum masses of the spin-$\frac{3}{2}$ baryons. The coupling of the baryon resonances to the spin-1 mesons will be discussed later. These new parameters will not influence the results for normal nuclear matter and finite nuclei.

B. Extrapolation to high densities

Once the parameters have been fixed to nuclear matter at ρ_0, the condensates and hadron masses at high baryon densities can be investigated, assuming that the change of the parameters of the effective theory with density are small. The behaviour of the fields and the masses of the baryon octet have been investigated in [?]. It is found that the gluon condensate χ stays nearly constant when the density increases. This implies that the approximation of a frozen glueball is reasonable. In these calculations the strange condensate ζ is only reduced by about 10 percent from its vacuum expectation value. This is not surprising since there are only nucleons in the system and the nucleon–ζ coupling is fairly weak. The main effect occurs for the non–strange condensate σ: This field drops to 30 percent of its vacuum expectation value at 4 times normal nuclear density, at even higher densities the σ field saturates. The behaviour of the condensates is also reflected in the behaviour of the baryon masses: The change of the scalar fields causes a change of the baryon masses in the dense medium. Furthermore, the change of the baryon masses depends on the strange quark content of the baryon. This is due to the different coupling of the baryons to the non-strange and strange condensate. The masses of the vector mesons are shown in fig. 3. The corresponding terms in the lagrangean are discussed in [?]. These masses stay nearly constant when the density is increased.

Now we discuss the inclusion of baryonic spin-$\frac{3}{2}$ resonances. How do they affect the behaviour of dense hadronic matter? We consider the two parameter sets C_1 and C_2, which satisfactorily describe finite nuclei [?]. As stated above, the main difference between the two parameter sets is the coupling of the strange condensate to the nucleon and to the Δ. In C_2 this coupling is set to zero, while the nucleon and the Δ couple to the ζ field in the case of C_1. Fig. 1 shows how the strength of the coupling of the strange condensate to the nucleon and the Δ depends on the vacuum expectation value of the strange condensate ζ_0. ζ_0 in turn is a function of the kaon decay constant ($\zeta_0 = \frac{1}{\sqrt{2}}(f_\pi - f_K)$). The results are obtained by changing the value of f_K, starting from parameter set C_1. f_K is expected to be in the range of 105 to 125 MeV [?]. For infinite nuclear matter one obtains good fits for the whole range of expected values. But when these parameter sets are used to describe finite nuclei, satisfactory results are only obtained for a small range of values for f_K, as can be seen for the proton single particle levels in fig. 2: with decreasing f_K the gap between the single-particle levels $1h_{\frac{9}{2}}$ and $3s_{\frac{1}{2}}$ in ^{208}Pb decreases such that e.g. for $f_K = 112 MeV$ the experimentally observed shell closure cannot be reproduced in the calculation. This result is not very surprising, because the smaller value of f_K leads to a stronger coupling of the nucleon to the strange field, with a mass of $m_\zeta \approx 1 GeV$. But it has been shown [?], that for a reasonable description of finite nuclei the nucleon must mainly couple to a scalar field with $m \approx 500 - 600 MeV$. The equation of state of dense hadronic matter for vanishing strangeness is shown in Fig. 4. Here two C_1 fits are compared, one with $f_k = 122$, which corresponds to the fit that has been tested to describe finite nuclei satisfactory in [?], and a C_1-type fit with $f_k = 116$, as the minimum acceptable value extracted from Fig. 2. The resulting values of coupling-constants to the nucleon are $g_{N\zeta} \approx 0.49$ for $f_K = 122$ MeV and $g_{N\zeta} \approx 1.72$ for $f_K = 116$ MeV. For the Δ-baryon $g_{\Delta\zeta} \approx -2.2$ and $g_{\Delta\zeta} \approx -0.59$, respectively. If these values are compared to the couplings to the non-strange condensate (which is around

-10 for the nucleon and the Δ in both cases) one observes that the mass difference between nucleon and Δ is due to the different coupling to the strange condensate.

Furthermore the resulting equation of state for parameter set C_2 is plotted. Here the nucleon and Δ-mass do not depend on the strange condensate. Fig. 4 shows two main results: The resulting EOS does not change significantly if f_K in the C_1-fits is varied within the reasonable range discussed above. In the following we refer to the C_1-fit of [**?**] with $f_K = 122 MeV$.
However, the different ways of nucleon and Δ mass generation lead to drastic differences in the resulting equations of state:

A pure σ-dependence of the masses of the nonstrange baryons (C_2) leads to an equation of state which is strongly influenced by the production of resonances at high densities. This is not the case when both masses are partially generated by the strange condensate (C_1), Fig. 4. In both fits the coupling of the Δ to the ω-meson ($g_{\Delta\omega}$) has been set equal to $g_{N\omega}$. The very different behaviour of the EOS can be understood from the ratio of the effective Δ-mass to the effective nucleon-mass, Fig. 5. If the coupling of the nucleon to the ζ field is set to zero (C_2), the mass ratio stays at the constant value $\frac{m_\Delta}{m_N} = \frac{g_{\Delta\sigma}}{g_{N\sigma}} \approx 1.31$. However, if the nucleon couples to the strange condensate (C_1), the mass ratio $\frac{m_\Delta}{m_N}$ increases with density, due to the different coupling of the nucleon and the Δ to the strange condensate ζ. The Δ does not feel less scalar attraction - the coupling to the σ field is the same for the nonstrange baryons. However, the mass of the Δ does not drop as fast as in the case of pure σ-couplings, and hence the production of baryon resonances is less favorable at high densities, Fig. 6.

Both coupling constants of the Δ-baryon are freely adjustable in the RMF models [**?**,**?**,**?**]. In the chiral model, which incorporates dynamical mass generation, the scalar couplings are fixed by the corresponding vacuum masses. If explicit symmetry breaking for the baryon mass generation is neglected, then the scalar couplings are fixed by the vacuum alone. To investigate the influence of the coupling to the strange condensate ζ, small explicitly symmetry breaking terms $m1, m2$ are used. This model behaves similar as the RMF models with $r = \frac{g_{\Delta\sigma}}{g_{N\sigma}} = \frac{m_\Delta}{m_N}$.

The remaining problem is the coupling of the resonances to the vector mesons. The coupling constants can be restricted by the requirement that resonances are absent in the ground state of normal nuclear matter. Furthermore possible secondary minimua in the nuclear equation of state should lie above the saturation energy of normal nuclear matter.

QCD sum-rule calculations suggest [?] that the net attraction for Δ‘s in nuclear matter is larger than that of the nucleon. From these constraints a 'window' of possible parameter sets $g_{\Delta\sigma}, g_{\Delta\omega}$ has been extracted [?]. In the chiral model one then obtains for each type of mass generation only a small region of possible values for $g_{\Delta\omega}$. The $\Delta - \omega$ coupling in Fig.7 is in this range. Pure σ-coupling (C_2) of the non-strange baryons yields a range of coupling constants $r_v = \frac{g_{\Delta\omega}}{g_{N\omega}}$ between $0.91 < r_v < 1$. For a non-vanishing ζ-coupling one obtains $0.68 < r_v < 1$. A smaller value of the ratio r_v (less repulsion), leads to higher Δ-probabilities and to softer equations of state. Due to this freedom in the coupling of the resonances to the vector mesons the equation of state cannot be predicted unambigiously from the chiral model. Here additional input from experiments are necessary to pin down the equation of state.

Finally we address the question, whether at very high densities the anti-nucleon potentials become overcritical. That means the potential for anti-nucleons may become larger than 2 $m_N c$ and nucleon- anti-nucleon pairs may be spontaneously emitted [?]. The nucleon and anti-nucleon potentials in the chiral model are shown as function of density (Fig. 8) for parameter set C_1 with and without quartic vector self-interaction. The latter is to obtain reasonable compressibility in the chiral model [?] and is in agreement with the principle of naturalness stated in [?]. From that the anti-nucleon potentials are predicted not to turn overcritical at densities below $12\rho_0$ in the chiral model (Fig. 8 left). Earlier calculations in RMF-models [?] did not include the higher order vector self-interactions. Then spontaneous anti-nucleon production occurs around $4 - 6\rho_0$. This also happens in the chiral model if the quartic-terms would be neglected (Fig. 8 right). The critical density shifts to even higher values, if the equation of state is softened by the baryon resonances, as can be seen in Fig. 9.

Hence, the chiral mean field model does not predict overcriticality for reasonable densities.

VI. CONCLUSION

Spin-$\frac{3}{2}$-baryon resonances can be included consistently in the nonlinear chiral SU(3)-model. The coupling constants of the baryon resonances to the scalar mesons are fixed by the vacuum masses. Two different ways of mass generation were investigated. It is found that they lead to very different predictions for the resulting equation of state of non-strange nuclear matter. The coupling of the baryon resonances to the vector mesons cannot be fixed. The allowed range of this coupling constant is restricted by requireing that possible density isomers are not absolutely stable, that there are no Δ's in the nuclear matter ground state and by QCD sum-rule induced assumption that the net attraction of Δ's in nuclear matter is larger than that for nucleons. Nevertheless, the behaviour of non-strange nuclear matter cannot be predicted unambigiously within the chiral $SU(3)$-model, so that further experimental input on Δ-production in high density systems and theoretical investigations on how the resonance production influences the observables in these systems (neutron stars, heavy ion-collisions) is needed. For both cases calculations are under way [**?**,**?**,**?**].

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APPENDIX A

The SU(3) matrices of the hadrons are (suppressing the Lorentz indices)

$$X=\frac{1}{\sqrt{2}}\sigma^a\lambda_a=\begin{pmatrix}(a_0^0+\sigma)/\sqrt{2} & a_0^+ & \kappa^+\\ a_0^- & (-a_0^0+\sigma)\sqrt{2} & \kappa^0\\ \kappa^- & \overline{\kappa^0} & \zeta\end{pmatrix}$$

$$P=\frac{1}{\sqrt{2}}\pi_a\lambda^a=\begin{pmatrix}\frac{1}{\sqrt{2}}\left(\pi^0+\frac{\eta^8}{\sqrt{1+2\,w^2}}\right) & \pi^+ & 2\,\frac{K^+}{w+1}\\ \pi^- & \frac{1}{\sqrt{2}}\left(-\pi^0+\frac{\eta^8}{\sqrt{1+2\,w^2}}\right) & 2\,\frac{K^0}{w+1}\\ 2\,\frac{K^-}{w+1} & 2\,\frac{\overline{K}^0}{w+1} & -\frac{\eta^8\sqrt{2}}{\sqrt{1+2\,w^2}}\end{pmatrix} \tag{A1}$$

$$V=\frac{1}{\sqrt{2}}v^a\lambda_a=\begin{pmatrix}(\rho_0^0+\omega)/\sqrt{2} & \rho_0^+ & K^{*+}\\ \rho_0^- & (-\rho_0^0+\omega)/\sqrt{2} & K^{*0}\\ K^{*-} & \overline{K^{*0}} & \phi\end{pmatrix} \tag{A2}$$

$$B=\frac{1}{\sqrt{2}}b^a\lambda_a=\begin{pmatrix}\frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p\\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda^0}{\sqrt{6}} & n\\ \Xi^- & \Xi^0 & -2\frac{\Lambda^0}{\sqrt{6}}\end{pmatrix} \tag{A3}$$

for the scalar (X), pseudoscalar(P), vector (V), baryon (B) and similarly for the axial vector meson fields. A pseudoscalar chiral singlet $Y=\sqrt{2/3}\eta_0\,\mathbb{1}$ can be added separately, since only an octet is allowed to enter the exponential.

The notation follows the convention of the Particle Data Group (PDG), [?], though we are aware of the difficulties to directly identify the scalar mesons with the physical particles [?]. However, note that there is increasing evidence that supports the existence of a low-mass, broad scalar resonance, the $\sigma(560)$-meson, as well as a light strange scalar meson, the $\kappa(900)$ (see [?] and references therein).

The masses of the various hadrons are generated through their couplings to the scalar condensates, which are produced via spontaneous symmetry breaking in the sector of the scalar fields. Of the 9 scalar mesons in the matrix X only the vacuum expectation values of the components proportional to λ_0 and to the hypercharge $Y\sim\lambda_8$ are non-vanishing, and the vacuum expectation value $\langle X\rangle$ reduces to:

$$\langle X \rangle = \frac{1}{\sqrt{2}}(\sigma^0\lambda_0 + \sigma^8\lambda_8) \equiv \mathrm{diag}\ (\frac{\sigma}{\sqrt{2}}, \frac{\sigma}{\sqrt{2}}, \zeta), \tag{A4}$$

in order to preserve parity invariance and assuming, for simplicity, $SU(2)$ symmetry[1] of the vacuum.

[1]This implies that isospin breaking effects will not occur, i.e., all hadrons of the same isospin multiplet will have identical masses. The electromagnetic mass breaking is neglected.

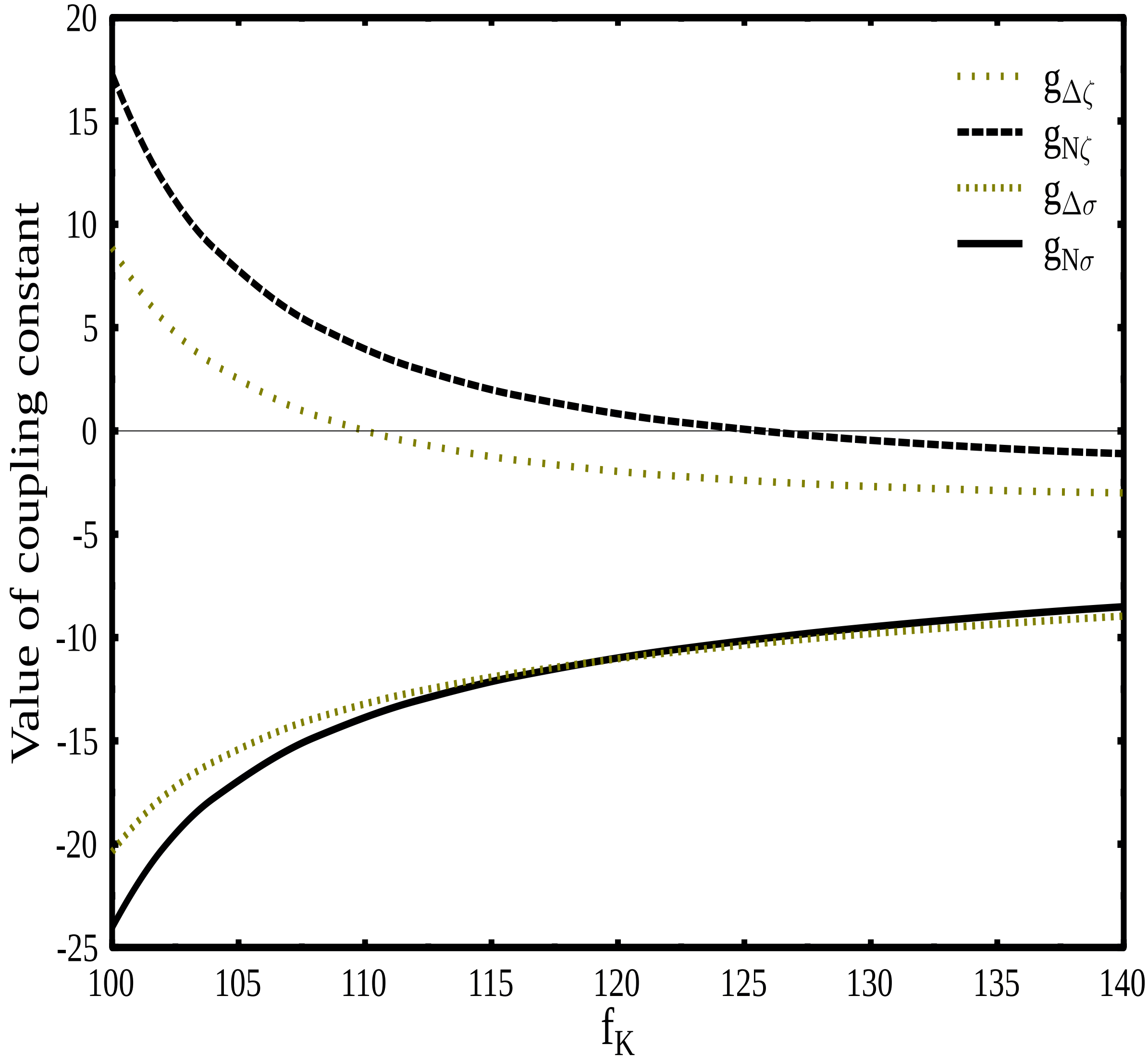

FIG. 1. Coupling of the nucleon and the Δ to the non-strange (σ) and strange (ζ) scalar condensates as a function of the kaon decay constant f_K. For $f_K \approx 115 - 125 MeV$ the coupling of the nucleon and the Δ to the non-strange scalar field σ are nearly equal. The coupling strength to the strange scalar field are different in sign. This results in the mass difference of $\Delta m \approx 300$MeV in the vacuum.

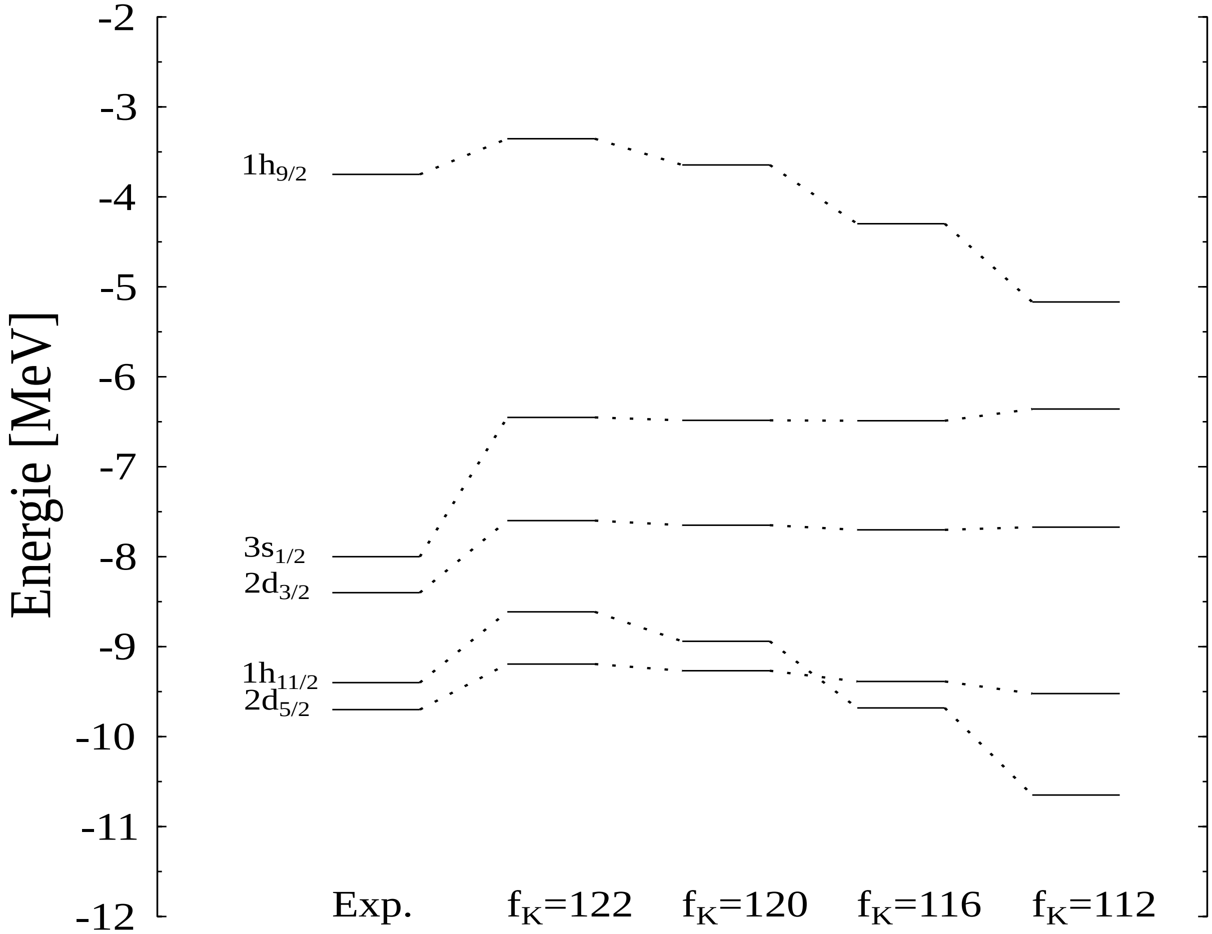

FIG. 2. Single particle energie levels for protons in Pb^{208} for parameter sets with different values of the kaon decay constant f_k (in MeV). The smaller the decay constant is chosen, the more the gap between the levels $1h_{\frac{9}{2}}$ and $3s_{\frac{1}{2}}$ decreases. The parameter sets were obtained by just variing f_k in C_1.

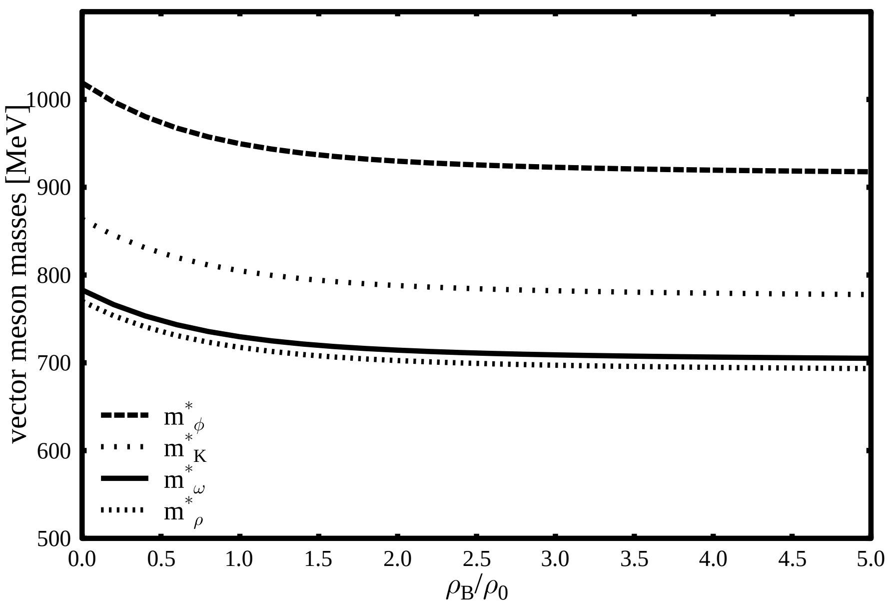

FIG. 3. Vector meson masses as a function of density.

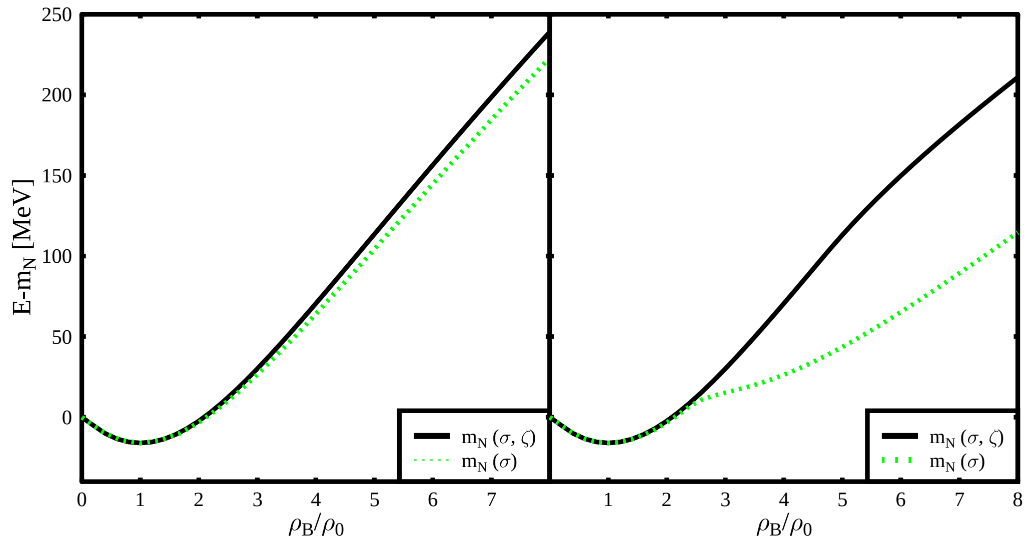

FIG. 4. Equation of state for infinite nuclear matter for the parameter sets C_1 ($m_N = m_N(\sigma, \zeta)$) and C_2 ($m_N = m_N(\sigma)$). In the left picture resonances are neglected while they are included in the right picture. If the strange condensates couples to the nucleon the influence of the Δ resonances on the equation of state is much weaker.

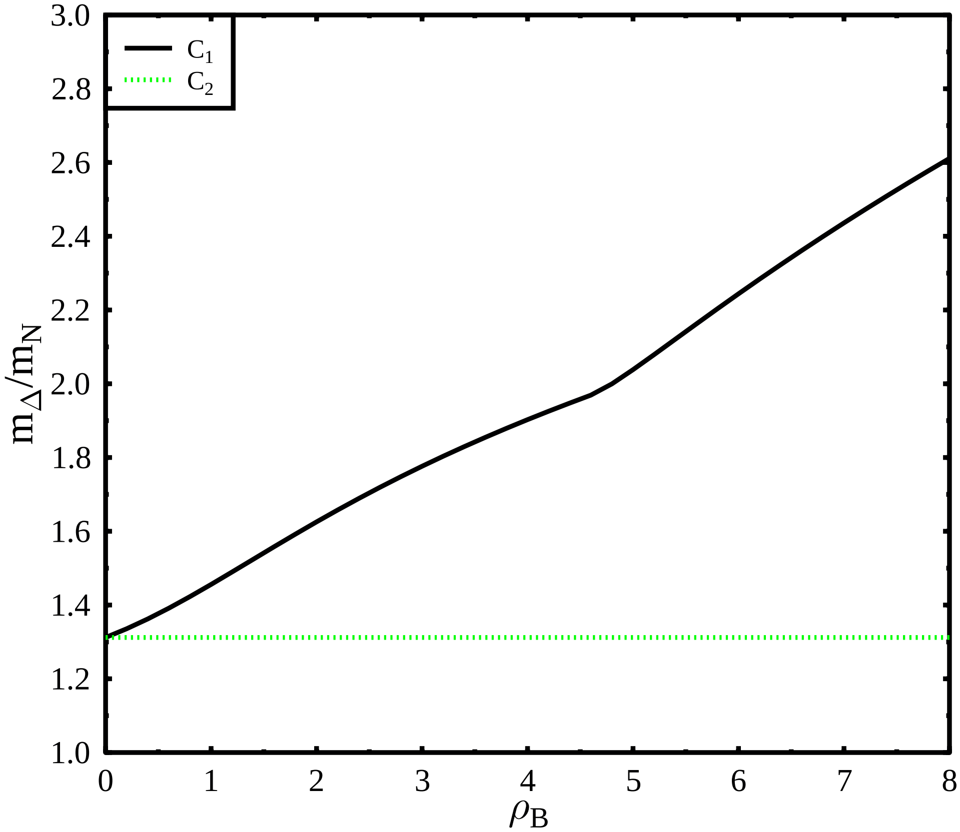

FIG. 5. Ratio of Δ-mass to nucleon-mass as a function of density for the two parameter sets C_1 and C_2.

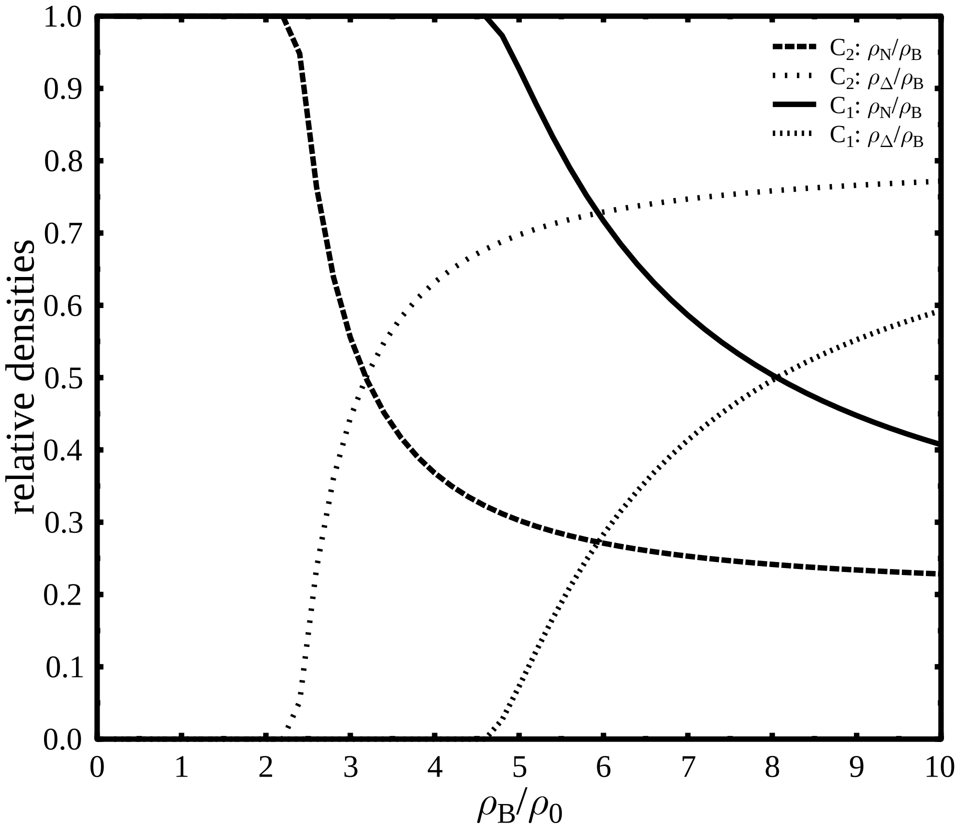

FIG. 6. Relative densities of nucleons and Δ's for various parameter sets. The production rate of Δ's depends strongly on the parameter set, i.e. on the strength of the nucleon-ζ and Δ-ζ coupling.

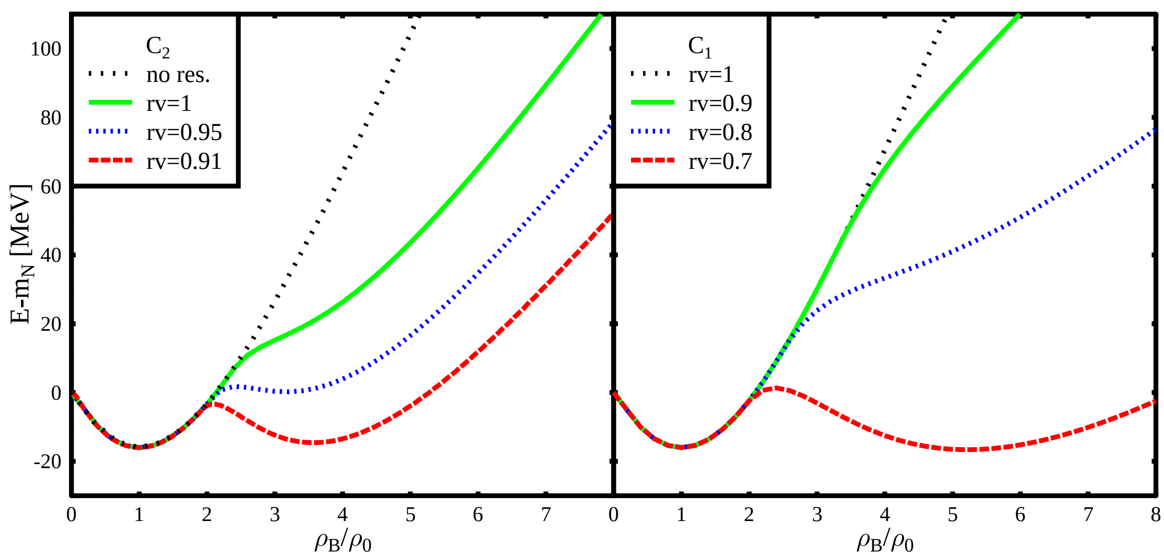

FIG. 7. Equation of state for parameter sets C_1 and C_2 for different values of the quotient $r_v = \frac{g_{N\omega}}{g_{\Delta\omega}}$. For the C_2-fit the value of r_v should not be less than 0.91 to avoid the density isomer being absolutely stable. For C_2, r_v must be larger than 0.68.

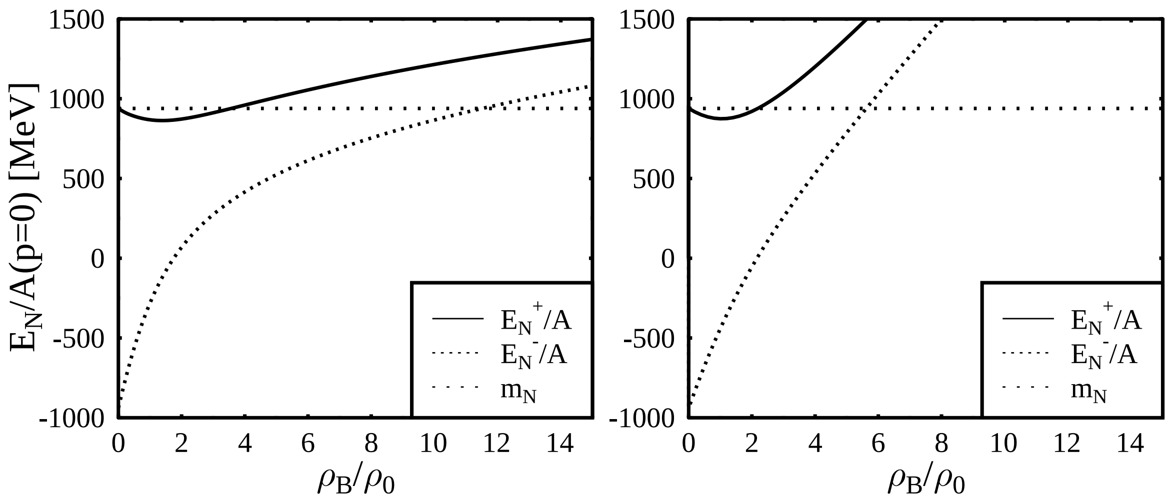

FIG. 8. Nucleon and anti-nucleon energy at $\vec{p} = 0$ as a function of baryon density. On the left hand side parameter set C_1 was use, while on the right hand side the coupling constant g4 for the quartic vector meson interaction was set to zero.

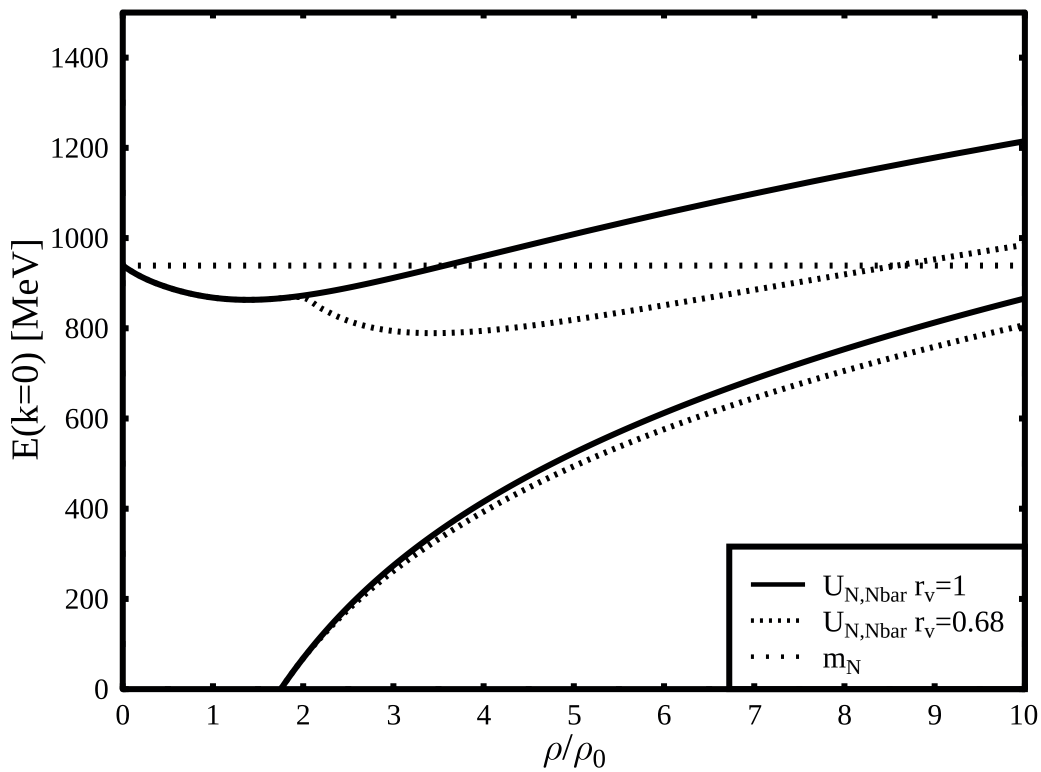

FIG. 9. Nucleon and anti-nucleon energy at $\vec{p} = 0$ as a function of baryon density using paramter set C_1 with $r_v = 1$ and $r_v = 0.68$. Lower values of the $\Delta - \omega$-coupling lead to a significant change of the equation of state (see fig.7) and to an increase of the critical density to even higher values.